
IN-CLASS ACTIVITY : LIMITS OF QUOTIENTS

1. Compute the following limits :

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| i) $\lim_{x \rightarrow 4} \frac{x^2-16}{x-4}$ | viii) $\lim_{x \rightarrow 3} \left(\frac{1}{x-3} - \frac{4}{x^2-2x-3} \right)$ |
| ii) $\lim_{x \rightarrow 2} \frac{x-2}{x^2-2x}$ | ix) $\lim_{x \rightarrow 5} \frac{\sqrt{x-1}-2}{x-5}$ |
| iii) $\lim_{x \rightarrow 6} \frac{3x-18}{2x-12}$ | x) $\lim_{x \rightarrow -3} \frac{\sqrt{x+4}-1}{x+3}$ |
| iv) $\lim_{x \rightarrow 9} \frac{x-9}{\sqrt{x}-3}$ | xi) $\lim_{x \rightarrow -2^-} \frac{2x^2+7x-4}{x^2+x-2}$ |
| v) $\lim_{x \rightarrow \pi} \frac{\sin(x)}{\tan(x)}$ | xii) $\lim_{x \rightarrow -2^+} \frac{2x^2+7x-4}{x^2+x-2}$ |
| vi) $\lim_{x \rightarrow 1} \frac{x^3-1}{x^2-1}$ | xiii) $\lim_{x \rightarrow 1^-} \frac{2x^2+7x-4}{x^2+x-2}$ |
| vii) $\lim_{x \rightarrow \frac{1}{2}} \frac{2x^2+3x-2}{2x-1}$ | xiv) $\lim_{x \rightarrow 1^+} \frac{2x^2+7x-4}{x^2+x-2}$ |

2. The density of an object is given by its mass divided by its volume $\rho = \frac{m}{V}$.

- Write the volume as a function $V(\rho)$ of the density.
- Assuming $m = 8kg$, plot the volume $V(\rho)$ for $0 < \rho < 80$.
- Evaluate $\lim_{\rho \rightarrow 0^+} V(\rho)$ and explain its physical meaning.

3. Sketch the graph of a function $f(x)$ with the following properties :

- f is defined for all real numbers ;
- $f(-6) = 3$;
- $\lim_{x \rightarrow -3^-} f(x) = \lim_{x \rightarrow -3^+} f(x) = 2$;
- $f(-3) = 3$;
- $\lim_{x \rightarrow 3^-} f(x) = f(3)$;
- f is not continuous at $x = 3$;
- $\lim_{x \rightarrow 0} f(x) = +\infty$.

4. Decide whether the following statements are true or false :

- The function $f(x) = \frac{2}{e^x - e^{-x}}$ is continuous everywhere.
- If $\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^-} f(x)$, then f is continuous at $x = 0$.
- The function $f(x) = \frac{x^2-4x+3}{x^2-1}$ is continuous for $2 \leq x < +\infty$.
- If a function is not continuous at a point, then it is not defined at that point.